Role Of History Of Mathematics To Improvement Of Education Of Mathematics

Saeed Seyed Agha Banihashemi
ihusaied2001@yahoo.com
School of International Relations
Tehran, Iran

***Abstract*—Question is whether we can teach mathematical analysis better by knowing history of mathematical analysis ? When mathematical analysis started and who invented? In this article I am answering these questions. In first part we have a look to history of mathematical Analysis and then a look to history of Infinity which is a important part of mathematical analysis.**

Introduction :

These days especially last 20 years ago this idea came to discussion that with help of history of mathematics we can teach better since we can tell to student when the subject started and who invented the subject .We can talk about time line of subject[index1]. The mathematician helped to development .By this way we can motivate the classes.

1-History of mathematical analysis :

Analysis is that branch of mathematics which talk about real numbers and complex number and their function s. It talk about formulation of calculus, continuity, integration and differentiability in general.

Analysis invented in the 17th century by invention of calculus by Newton and Leibniz. . In the 17th and 18th centuries analysis topics such calculus of variation , differential and partial differential equation , Fouire analysis and generating function were developed mostly in applied work .Calculus techniques were applied successfully to approximation discrete problems by continues ones. All through the 18th century the definition was a subject of debate among mathematicians. In the 19th century, Cauchy was the first to put calculus on an exact foundation by introducing the concept of Cauchy sequence. He also started the formal theory complex analysis. Poission , Liouville , Fourier and others studied partial differentail equations and harmonic analysis.

In the middle of the century Riemanian introduced his theory of intigration . The last third of the 19th century saw the arithmetization of analysis by Weierstrass ,who thought that geometric reasoning was inherently misleading, and introduced the $\varepsilon-\delta$ definition of limit .

. Then mathematicians started worrying that they were assuming the existence of a continuum of real numbers by Dedkind cuts. Around that time the attempts to refine the theorem of Riemanian integration led to the study of the size of the discontinuity sets of real functions.

Also monsters "(nowhere continues function s, continuous but nowhere differentiable functions, space filling curves) began to be created. In this context, Jordan developed his theory of measure, Cantor developed what is now called naïve set theory, and Baier proved the Baire category theorem. In the early 20th century, calculus was formalized using set theory. Lebesue solved the problem of measure , and Hilbert introduced Hilbert space to solve integral equations. The idea of normed vector space was in the air and in the 1920 Banch created functional analysis. You can see the diagram of development of mathematical analysis in the index [index-1].
Now these questions arise the person want to teach analysis is better to know this history or not?

2- History of Infinity

We can talk about history of calculus, continuity, integration, differentiability, Infinity, …… but it will be a big article since Infinity is very important in Analysis we concentrate on history of Infinity.
2-1- We could not find any manuscript before Greek about Infinity for first time Aristotle in mathematical calculation used apeiron which in Greek language means not defined when he reach to $x^2+1=0$ use word apeiron. In the Pythagorean work they came to $\sqrt{2}$,we could not find anything about infinity because they thinking about numbers only. In famous book of Euclid's (elements) when he wants to define a line express as follow it can be extended as far as necessary. In Indian and Islamic mathematics we face same manner.
In more practical direction, Fibonacci linked irrational number with Infinity by working on $\sqrt{\sqrt{a}\pm\sqrt{b}}$ where a,b are rational he tell because we cannot touch irrational number so they are under cloud of Infinity. Johan Wallis one of most important of 17th century mathematician for first time introduced the symbol of ∞ for infinity .By invention of new calculus by Newton, Leibnitz, Euler, Bernoulli , Infinity became so important. Let us review the way Newton explained derivative of x^2 :

$$(x+\sigma)^2 - x^2 = x^2 + 2x\sigma + \sigma^2 - x^2$$
$$= 2x\sigma + \sigma^2$$
$$\Rightarrow \frac{2x\sigma + \sigma^2}{\sigma} = 2x + \sigma$$

Eliminate σ so we have $2x$ he call this as moment.[1]

The great Leonard Euler (1707-1783)did not improve the theory state of affairs at all. He pursued new analysis with an abandon that would have cautioned even Newton and Leibnitz follow his way ,he considered two series

$$\frac{1}{(x+1)^2} = 1 - 2x + 3x^2 - 4x^3 .... \quad *$$
$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ... \quad **$$

He puts x=-1 in $*$ then $\infty = 1+2+3+.....$.

And put x=2 in $**$ then -1=1+2+4+……… so $-1 > \infty$ And if x=-1 in $**$ then $\frac{1}{2} = 1-1+1-1+1-....$ this kind of computation those days in analysis called paradoxes. Euler called $\frac{0}{0}$ infinity.

Fourier(1768- 1830) applied trigonometric to solve heat problem.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$
$$b_n = \frac{2}{\pi} \int_0^{\pi} f(s) \sin nsds$$

Fourier stated that every function could be expressed as trigonometry function. Next question was Classification of function s for which Fourier series converge. This question was very important for development of analysis, which in the end set theory, functional analysis invented and reorganization analysis.[2]

Gorge Cantor (1845-1918) he study families of functions having convergence Fourier series as classified by exceptional points. This was an answer of Riemann's question .Let us see details:

Cantor most certainly was aware that the process of derivation could be carried out indefinitely. Use the notation S^n to be nth derived set of S .Then $(S)^{n+1} = (S)^n)'$, the derived set of S^n for every n.If we continue operation we have .

$$S^0, S^1, ...S^{\infty}, S^{\infty+1}, ...S^{\infty-2}, ..., S^{\infty^2}, ..., S^{\infty^{\infty}}, ...$$

the number ∞ is naturally appearing in this context. He tried to much to distinguish the set of real and irrational.

In 1874 he established that the set of algebraic number could be put into one to one correspondence with natural number. But set of real number cannot be put into a correspondence.[3]

Theorem: The set of rational numbers is one-to-one correspondence with natural number.

Proof: Let $r_{m,n} = \frac{m}{n}$ be a a rational number in the reduced form .Define rational as

$$r_{m,n} \to 2^m 3^n$$

This give correspondence of rational to a sub set of rational numbers and hence to the natural numbers. Now by task of Cantor Infinity is now a number in its own right it is linked with counting ideas and relations to sets of sets.

In 1882 Cantor introduced a new infinity, distinguishing cardinality from order, cardinal number from ordinal number. He say that $(a_1, a_2, a_3, ...)$ and $(b_2, b_3, b_4, ...b_1)$ have same cardinality but their order is different .First have order ω and second $\omega + 1$ .For finite sets there is only one order even though elements can be transposed. So ordinal and cardinal number can be identified. Cantor showed how to produce a set with a power greater than natural numbers, namely the set of ordinal numbers of the power of natural numbers.

Form this he went to construct power set of the set of ordinals, and so on generating higher and higher powers. Now to make contact with the power of real numbers cantor made assumption real are well ordered, which is defined below. From this he established that the power of real numbers is less than or equal greater than each of the new power but not which of them it is. [4]

By 1895 cantor defined exponentiation. Using the term $\aleph_0$ t0 denote the cardinality of natural numbers, he defined $2^{\aleph_0}$ for cardinality of real. $\aleph_1$ The next large cardinal than $\aleph_0, \aleph_\omega$ denote ω Th cardinal number the continuum hypothesis is written as $2^{\aleph_0} = \aleph_1$.

Cantor and others produced similar examples of a special category of nowhere-dense sets as an application arose of these idea .The special new category consist of those that are fat in the following way: Every finite covering of the set by interval should have total length greater than some given number ,say 1.It becomes natural to say that such sets have *content,* which had important role to development of modern integral, notably the Jordan completion to the Cauchy -Rieman integral and ultimately the L.ebesgue integral. So we see here set and infinty now giving rise to new idea for analysis .And note that the Fourier series problem that served as the root of these investigation would find its ultimate solution within the context of the modern integral.

In 1873 a French mathematician named Paul Reymond (1831-1889) discovered a continuous for which its Fourier series diverged at a single point by using three following theorem :

Theorem:. If $f(x)$ is continues on $[0,\pi]$ then $S_m(f,x) \to f(x)$ for all $t \notin E$

Where E is some set of measure zero.Here

$$S_m(f,x) = \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

[]

Theorem: (khane and Katzelson). If E is a set of measure zero then there is continuous function $f(x)$ on $[0,\pi]$ for which $\limsup_{m\to\infty} S_m(f,x) = \infty$ for all $x \in E$.

Theorem (Kolmogorov). There is a Lebsegue inferable function F(x) whose Fourier series diverges at every point.[5]

3-Axiom of choice :

In 1904 Zermelo formulated axiom of choice when he published this axiom he faced by detractions from Borel and Bernstine the kernel of their argument was :" unless a definite law specified which element was chosen from each set no real choice has been made and the new set was not relay formed. Borel called this theorem lawless choice . He had supporter like Jacques Hadamard (1865-1963).

But we know that axiom of choice these days widely used wide and varied results have been derived from it.

The *Well ordering Axiom of choice* was used widely by Cantor and going to be used these days .

" If a set is linearly ordered then there is a relation < for which for every $a,b \in M$ then a<b ,b<a or a=b is true . The set is well ordered if every sub set of M has a least element ".

For example natural number is well ordered but real are not .Zermelo use axiom of choice to answer a lot of question he used axiom of choice to well- ordering of real.

4-How large is Infinity?

We know that by power set construction we can construct large cardinals again and again how much we can go?

It can be shown that for each set M of cardinals there is smallest cardinal succeeding all member of M.Denote this cardinal by SupM.

For example $N_0 = Sup\{0,1,2,3,....\}$ $N_1 = Sup\{N_0\}$ .A cardinal A which are not zero is said to be *inaccessible* if

1- for every set *B* of cardinals such that $|B| \prec A$, Sup*B<A.*

2- if C<A, $C \in A, 2^C \prec A$.

Very easy one can check N_0 is inaccessible. What about the others?[5]

6- Conclusion: In this way who can teach analysis better with above information or without above information ? Any logic mind prefer first one.

References:

[1]- Crumps ,T.1997 . *The Anthropology of Numbers-* Cambridge university.

[2]-Bourbaki,N.1999. *Elements of the History of Mathematics-* Springer.

[3]-Fauvel,J-Gray,J.1987 . *The History of mathematics* - Open University.

[4]-Chambert,J- 1993. *A History of Algorithms-* Springer.

[5]-Cheverghese ,J.1994. *Non-European Roots of Mathematics*-Penguin